高瞻計畫_振動學課程
Lecture 6: Multiple Degrees of Freedom Systems (II)

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Outline

- Influence coefficients
- Modal matrix
- Lagrange's Equations
- Solution schemes
- Finite element method, an introduction
- Simple problems
- Demonstrations

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Introduction

- This lecture presents several key issues
  - How to convert physical system to MDOF vibration model
    - Influence coefficient methods
  - Modal matrix and vibration modes
  - Energy method to form the equations of motion
    - Lagrange's equation
  - Approximate methods
    - Approximation methods and finite element method

Part I. Influence Coefficients

Introduction

- Matrix methods for analysis provides systematic approach to solve complicate problem
- In addition, can also use them to predict key properties of vibration systems
  - Natural frequency
  - Semi-definite

Flexibility Matrix

- Consider a 3 DOF mass-spring system, under quasi-static situation, the force-displacement relation can be expressed as

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix}$$

or

$$\{x\} = [a]\{f\}$$

[a] = flexibility matrix

Stiffness Matrix

- On the other hand,

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

or

$$\{f\} = [k]\{x\}$$

[k] = stiffness matrix

$[k]=[a^{]-1}$; $[a]=[k]^{-1}$

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How to Obtain Influence Coefficients

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix}$$

First, let $f_1=1$, $f_2=f_3 = 0$ → $a_{11} = x_1$; $a_{12} = x_2$; $a_{13} = x_3$

then, let $f_2=1$, $f_1=f_3 = 0$ → $a_{21} = x_1$; $a_{22} = x_2$; $a_{23} = x_3$

Finally, let $f_3=1$, $f_1=f_2 = 0$ → $a_{31} = x_1$; $a_{32} = x_2$; $a_{33} = x_3$

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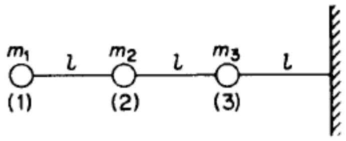
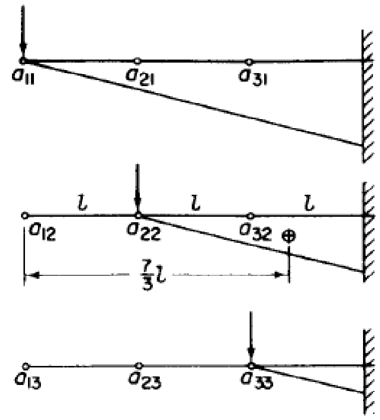
An Example

Determining the flexibility matrix of the following system





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From cantilever beam formula from mechanics of materials

$$a_{12} = \frac{1}{EI}\left[\frac{1}{2}(2l)^2 \times \frac{7}{3}l\right] = \frac{14}{3}\frac{l^3}{EI}$$

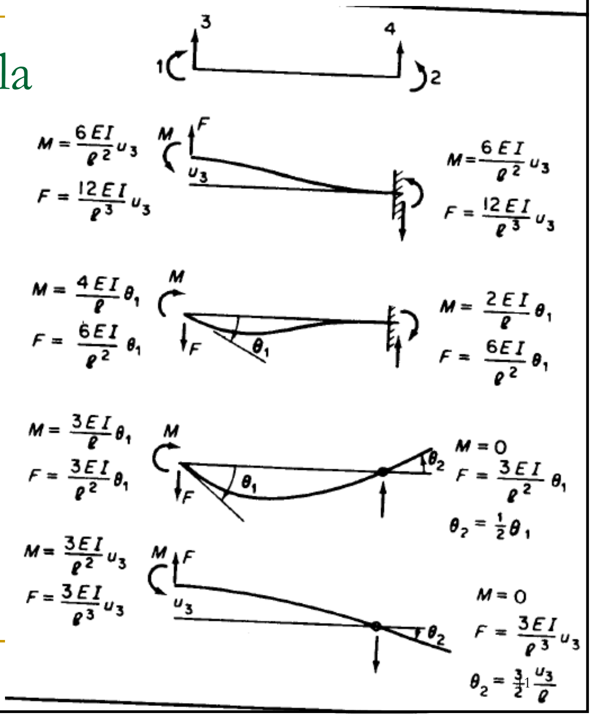
$$a_{11} = \frac{27}{3}\frac{l^3}{EI} \qquad a_{21} = a_{12} = \frac{14}{3}\frac{l^3}{EI}$$

$$a_{22} = \frac{8}{3}\frac{l^3}{EI} \qquad a_{23} = a_{32} = \frac{2.5}{3}\frac{l^3}{EI}$$

$$a_{33} = \frac{1}{3}\frac{l^3}{EI} \qquad a_{13} = a_{31} = \frac{4}{3}\frac{l^3}{EI}$$

$$a = \frac{l^3}{3EI}\begin{bmatrix} 27 & 14 & 4 \\ 14 & 8 & 2.5 \\ 4 & 2.5 & 1 \end{bmatrix}$$

Essential Formula



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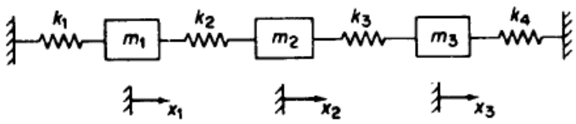
Stiffness Matrix Example

Find the stiffness matrix



Solution: Let $x_1 = 1.0$ and $x_2 = x_3 = 0$. The forces required at 1, 2 and 3, considering forces to the right as positive, are

$$f_1 = k_1 + k_2 = k_{11}$$

$$f_2 = -k_2 = k_{21}$$

$$f_3 = 0 = k_{31}$$

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Cont'd

Repeat with $x_2 = 1$, and $x_1 = x_3 = 0$. The forces are now

$$f_1 = -k_2 = k_{12}$$

$$f_2 = k_2 + k_3 = k_{22}$$

$$f_3 = -k_3 = k_{32}$$

For the last column of k's, let $x_3 = 1$ and $x_1 = x_2 = 0$. The forces are

$$f_1 = 0 = k_{13}$$

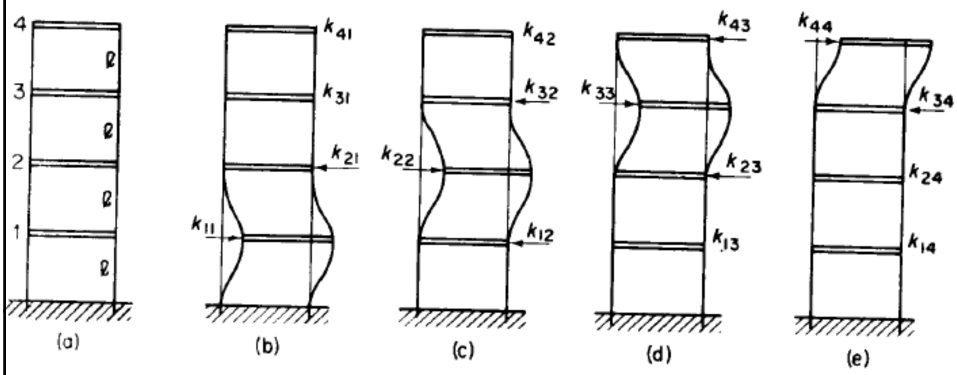
$$f_2 = -k_3 = k_{23}$$

$$f_3 = k_3 + k_4 = k_{33}$$

The stiffness matrix can now be written as

$$K = \begin{bmatrix} (k_1 + k_2) & -k_2 & 0 \\ -k_2 & (k_2 + k_3) & -k_3 \\ 0 & -k_3 & (k_3 + k_4) \end{bmatrix}$$

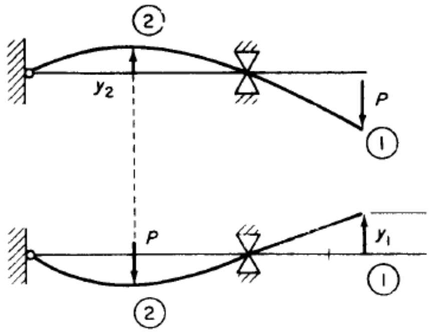
A Building Example



$$k_{11} = k_{22} = k_{33} = \frac{192EI}{(2l)^3} = 24\frac{EI}{l^3}$$

$$[k] = \frac{EI}{l^3}\begin{bmatrix} 24 & -12 & 0 & 0 \\ -12 & 24 & -12 & 0 \\ 0 & -12 & 24 & -12 \\ 0 & 0 & -12 & 12 \end{bmatrix}$$

Reciprocity Theorem



For an elastic structure, the displacement at 2 due to force applied at 1 = the displacement at 1 due to force applied at 2

This implies that stiffness or flexibility matrix are symmetric
and the required test numbers can be cut in half

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Characteristic Equations

$$M\ddot{X} + KX = 0$$

$$M^{-1}M = I \text{ (a unit matrix)}$$

$$M^{-1}K = A \text{ (a system matrix)}$$

Aka Dynamic Matrix

$$I\ddot{X} + AX = 0$$

Assuming harmonic motion $\ddot{X} = -\lambda X$, where $\lambda = \omega^2$,

$$[A - \lambda I]\{X\} = 0$$

Stiffness formulation

$$[am - \lambda I]X = 0, \quad \lambda = \frac{1}{\omega^2}$$

Flexibility formulation

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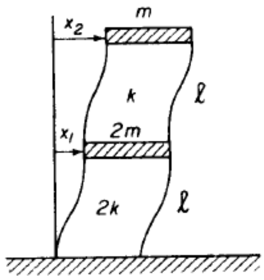
Orthogonality of Eigenvectors

- For different eigenvalues, eigenvectors are mutually orthogonal (perpendicular to each other)
  - Can be used as a new coordinates to describe the behavior of vibration systems
    - Principal coordinates
  - Can be used to solve vibration of continuous systems
    - PDE → series ODEs

Example: Thompson 6.3.1

Determining the natural frequencies and mode shapes of a 2-story building



$$\begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

letting $\lambda = \omega^2$

$$\begin{bmatrix} \left(\frac{3}{2}\frac{k}{m} - \lambda\right) & -\frac{k}{2m} \\ -\frac{k}{m} & \left(\frac{k}{m} - \lambda\right) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

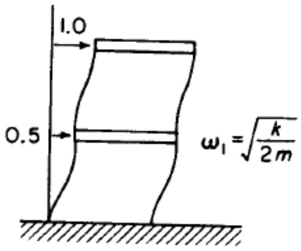
$$\lambda^2 - \frac{5}{2}\frac{k}{m}\lambda + \left(\frac{k}{m}\right)^2 = 0$$

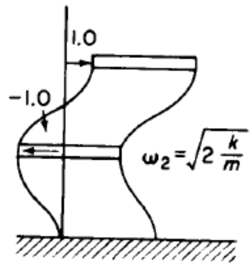
$$\lambda_1 = \frac{1}{2}\frac{k}{m}$$

$$\lambda_2 = 2\frac{k}{m}$$

Cont'd

$$X_1 = \begin{Bmatrix} 0.50 \\ 1.00 \end{Bmatrix} \qquad X_2 = \begin{Bmatrix} -1.00 \\ 1.00 \end{Bmatrix}$$





Part II. Modal Matrix

Modal Matrix Definition

- 3-DoF system example

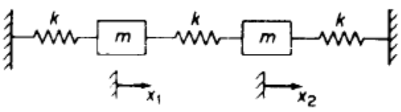
$$P = \left[\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}_1 \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}_2 \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}_3 \right] = [X_1\ X_2\ X_3]$$

$$P' = \begin{bmatrix} (x_1\ x_2\ x_3)_1 \\ (x_1\ x_2\ x_3)_2 \\ (x_1\ x_2\ x_3)_3 \end{bmatrix} = [X_1\ X_2\ X_3]'$$

$$P'MP = [X_1 \quad X_2]'[M][X_1 \quad X_2] = \begin{bmatrix} X_1'MX_1 & X_1'MX_2 \\ X_2'MX_1 & X_2'MX_2 \end{bmatrix} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}$$

$$P'KP = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$$

Example



$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \{0\}$$

$$\lambda_1 = \omega_1^2 = \frac{k}{m} \qquad \lambda_2 = \omega_2^2 = 3\frac{k}{m}$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}_{\lambda_1} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}, \quad \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}_{\lambda_2} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \qquad P = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\tilde{P}'M\tilde{P}\ddot{Y} + \tilde{P}'K\tilde{P}Y = 0$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \frac{k}{m} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = 0$$

Cont'd

- The system is decoupled

x_i: physical coordinates
y_i: principal coordinate

$$\ddot{y}_i + \omega_i^2 y_i = 0$$

$$y_i(t) = y_i(0)\cos\omega_i t + \frac{1}{\omega_i}\dot{y}_i(0)\sin\omega_i t$$

$$x_1(t) = \frac{1}{\sqrt{2m}}[y_1(t) - y_2(t)]$$

$$x_2(t) = \frac{1}{\sqrt{2m}}[y_1(t) + y_2(t)]$$

$$y_1(t) = \frac{1}{2}\sqrt{2m}\left\{[x_1(0) + x_2(0)]\cos\omega_1 t + \frac{1}{\omega_1}[\dot{x}_1(0) + \dot{x}_2(0)]\sin\omega_1 t\right\}$$

$$y_2(t) = \frac{1}{2}\sqrt{2m}\left\{[-x_1(0) + x_2(0)]\cos\omega_2 t + \frac{1}{\omega_2}[-\dot{x}_1(0) + \dot{x}_2(0)]\sin\omega_2 t\right\}$$

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Modal Damping

$$M\ddot{X} + C\dot{X} + KX = F$$

$$\tilde{P}'M\tilde{P}\ddot{Y} + \tilde{P}'C\tilde{P}\dot{Y} + \tilde{P}'K\tilde{P}Y = \tilde{P}'F$$

If Rayleigh Damping

$$C = \alpha M + \beta K$$

$$\tilde{P}'C\tilde{P} = \alpha\tilde{P}'M\tilde{P} + \beta\tilde{P}'K\tilde{P}$$
$$= \alpha I + \beta\Lambda$$

$$\ddot{y}_i + (\alpha + \beta\omega_i^2)\dot{y}_i + \omega_i^2 y_i = \tilde{f}_i(t)$$

$$2\zeta_i\omega_i = \alpha + \beta\omega_i^2$$

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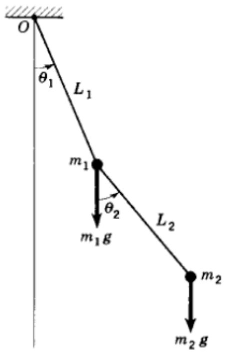
Part III. Lagrange's Equation

Introduction

- Perhaps Lagrange's equation is the most well-known method in dynamics (next to Newton's 2^{nd} Law)
- Energy approach
  - Scalar relation
- obtaining equations of motion by kinetic and potential energies

Step 1: Define Generalized Coordinates

- Usually requires "independent"
- Example
  - Double pendulum
  - Use θ_1, θ_2 as the generalized coordinates



$$x_1 = l_1 \sin\theta_1 \qquad x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$$
$$y_1 = l_1 \cos\theta_1 \qquad y_2 = l_1 \cos\theta_1 + l_2 \cos\theta_2$$

θ_1, θ_2: as q1, q2 (Generalized coordinates)

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Step 2. Find Kinetic and Potential Energies

$$v_1^2 = \dot{x}_1^2 + \dot{y}_1^2 = (l_1\dot{\theta}_1)^2$$

$$v_2^2 = \dot{x}_2^2 + \dot{y}_2^2 = \left[l_1\dot{\theta}_1 + l_2\dot{\theta}_2\cos(\theta_2 - \theta_1)\right]^2 + \left[l_2\dot{\theta}_2\sin(\theta_2 - \theta_1)\right]^2$$

$$T = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$V = -m_1(l_1\cos\theta) - m_2(l_1\cos\theta_1 + l_2\cos\theta_2)$$

Lagrangian L = T-V

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Step 3. Apply Lagrange's Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \qquad i = 1, 2, \cdots N$$

$$L = T - V = \tfrac{1}{2}(m_1 + m_2)(L_1\dot{\theta}_1)^2 + \tfrac{1}{2}m_2(L_2\dot{\theta}_2)^2 + m_2L_1L_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_2 - \theta_1) - (m_1 + m_2)L_1 g(1 - \cos\theta_1) - m_2L_2 g(1 - \cos\theta_2).$$

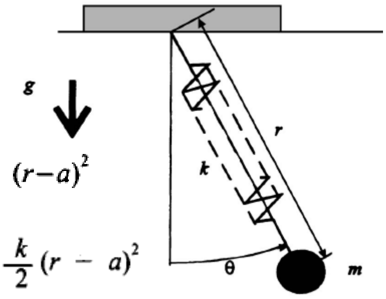
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = (m_1 + m_2)L_1^2\ddot{\theta}_1 + m_2L_1L_2\ddot{\theta}_2 \cos(\theta_2 - \theta_1) - m_2L_1L_2\dot{\theta}_2^2 \sin(\theta_2 - \theta_1) + (m_1 + m_2)L_1 g \sin\theta_1 = 0,$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = m_2L_2^2\ddot{\theta}_2 + m_2L_1L_2\ddot{\theta}_1 \cos(\theta_2 - \theta_1) + m_2L_1L_2\dot{\theta}_1^2 \sin(\theta_2 - \theta_1) + m_2L_2 g \sin\theta_2 = 0.$$

Step 4. Linearizing equations and form the vibration model

- Small amplitude analysis
  - Linearization
  - Obtain linear vibration model
- Analyzing the vibration model
  - Natural frequencies
  - Natural mode
  - Time response,….

Example: A 2-D Pendulum with Flexible String



Generalized Coordinates: θ, r

$$T = \frac{m}{2}\left[\dot{r}^2 + (r\dot{\theta})^2\right] \qquad V = -mgr\cos\theta + \frac{k}{2}(r-a)^2$$

$$\mathfrak{L} = T - V = \frac{m}{2}\left[\dot{r}^2 + (r\dot{\theta})^2\right] + mgr\cos\theta - \frac{k}{2}(r-a)^2$$

$$\frac{d}{dt}\left(\frac{\partial\mathfrak{L}}{\partial\dot{r}}\right) - \left(\frac{\partial\mathfrak{L}}{\partial r}\right) = Q_r$$

$$m\ddot{r} - mr\dot{\theta}^2 - mg\cos\theta + k(r-a) = 0$$

$$\frac{d}{dt}\left(\frac{\partial\mathfrak{L}}{\partial\dot{\theta}}\right) - \left(\frac{\partial\mathfrak{L}}{\partial\theta}\right) = Q_\theta = 0$$

$$2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta} - mgr\sin\theta = 0$$

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Part IV. Normal Mode Summation

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Introduction

- Solving MDOF vibration problem can be a pain and time consuming
  - As the # of DoF becomes larger
  - Unnecessary detail
- Approximation methods become popular
  - Obtain effective solution with much less computational cost
- Normal mode summation method
  - Aka mode decomposition, mode superposition
  - Effective and has been implemented in FEM

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Approach

- A method to solve force vibration response based on normal modes
- Vibration of a 50-story building: use first 3 modes to approximate

$$M\ddot{X} + C\dot{X} + KX = F$$

$$x_i = \phi_1(x_i)q_1(t) + \phi_2(x_i)q_2(t) + \phi_3(x_i)q_3(t)$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} = \begin{bmatrix} \phi_1(x_1) & \phi_2(x_1) & \phi_3(x_1) \\ \vdots & \vdots & \vdots \\ \phi_1(x_n) & \phi_2(x_n) & \phi_3(x_n) \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{Bmatrix}$$

ϕ_i: known mode shapes

q_i: unknown time-varying coefficients

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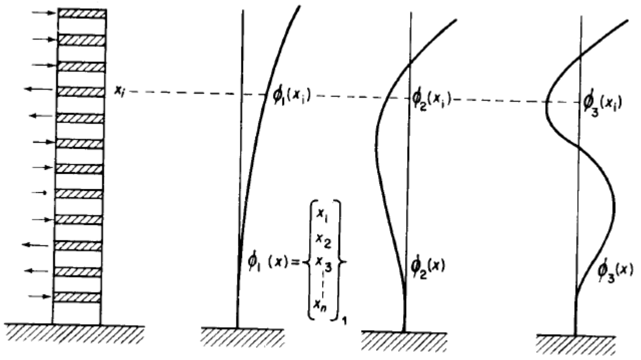

$$P'MP\ddot{q} + P'CP\dot{q} + P'KPq = P'F$$

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega_i^2 q_i = \Gamma_i f(t)$$

Mode participation factor

$$\Gamma_i = \frac{\sum_j \phi_i(x_j)p(x_j)}{\sum_j m_j\phi_i^2(x_j)}$$

Original: 50 DoFs
This approach: 3 DoFs

Part V. Finite Element Method: An Introduction

Introduction

- Finite element analysis is an effective and systematic method for modeling and analyzing various engineering problems
  - Deformation, vibration, fluid flow, electromagnetics, acoustic, etc…
  - Many commercial available packages
- Pre-processor
  - Obtain the equations of motion by meshing…
- Solver
  - Solve the matrix equation
- Post-processor
  - Visualizing the results

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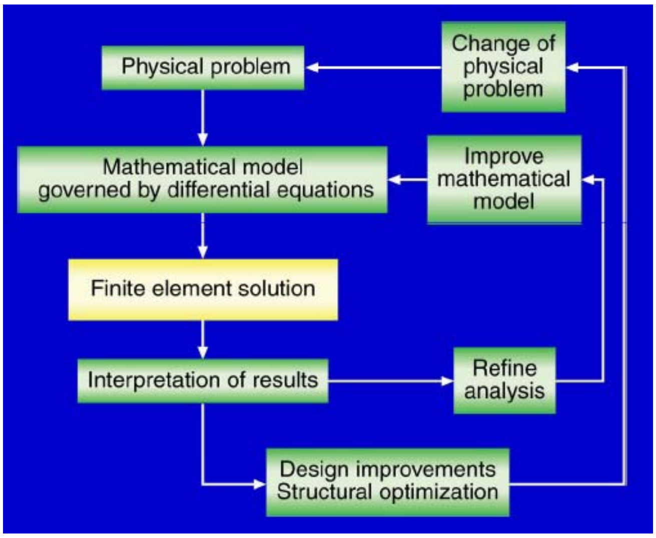
Finite Element Analysis Procedure



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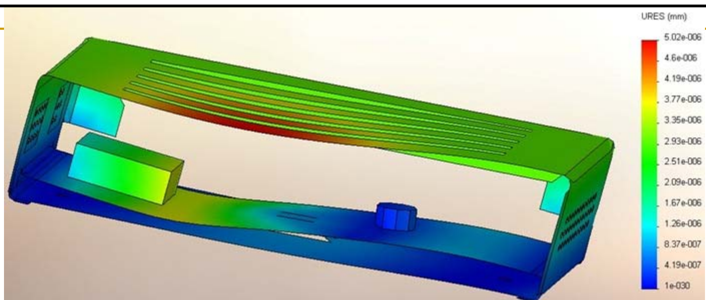
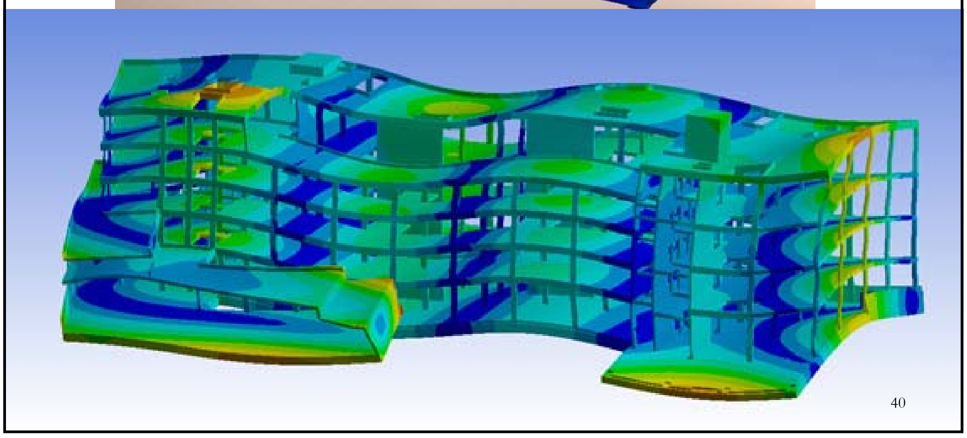
Typical Commercial FEM Packages for Vibrations

- ANSYS
- ABAQUS
- ADINA
- MSC/Nastran
- COMSOL





Part VI. Simple Problems

Prob. 1 Influence Coefficients (Rao 6.1)

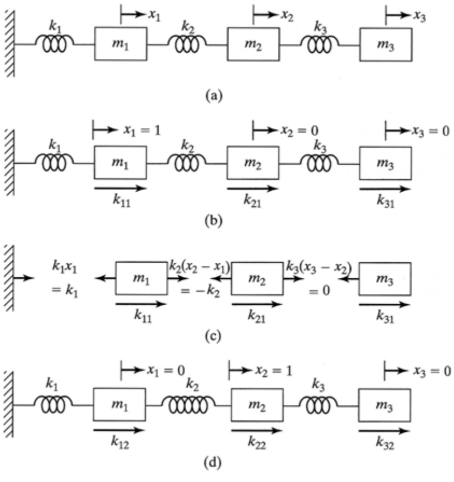


FIGURE 6.6 Determination of stiffness influence coefficients.

Find the stiffness influence coefficients of the system shown in Fig. 6.6(a).

Prob. 2 Stiffness Matrix of a Frame (Rao. 6.4)

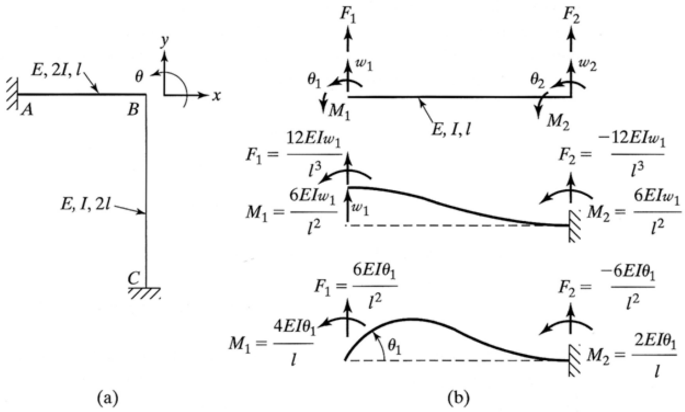
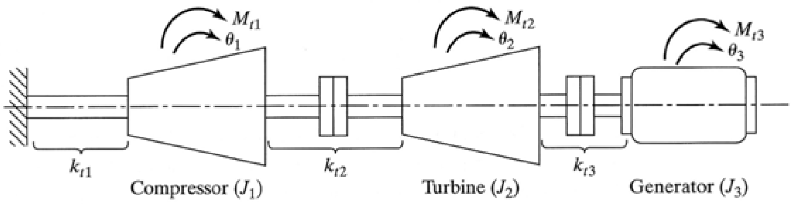


FIGURE 6.7 Stiffness matrix of a frame.

Determine the stiffness matrix of the frame shown in Fig. 6.7(a). Neglect the effect of axial stiffness of the members AB and BC.

Prob. 3 Equation of Motion of a Torsional System (Rao. 6.8)



The arrangement of the compressor, turbine, and generator in a thermal power plant is shown in Fig. 6.11. This arrangement can be considered as a torsional system where J_i denote the mass moments of inertia of the three components (compressor, turbine, and generator), M_{ti} indicate the external moments acting on the components, and k_{ti} represent the torsional spring constants of the shaft between the components, as indicated in Fig. 6.11. Derive the equations of motion of the system using Lagrange's equations by treating the angular displacements of the components θ_i as generalized coordinates.

Prob. 4 Forced Vibration Response of a Forging Hammer (Rao. 6.16)

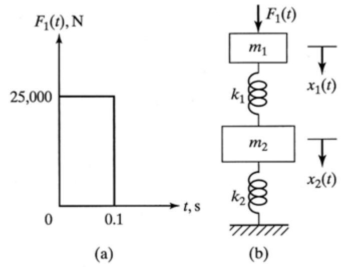


FIGURE 6.14 Impact caused by forging hammer.

The force acting on the workpiece of the forging hammer shown in Fig. 5.41 due to impact by the hammer can be approximated as a rectangular pulse, as shown in Fig. 6.14(a). Find the resulting vibration of the system for the following data: mass of the workpiece, anvil and frame $(m_1) = 200$ Mg, mass of the foundation block $(m_2) = 250$ Mg, stiffness of the elastic pad $(k_1) = 150$ MN/m, and stiffness of the soil $(k_2) = 75$ MN/m. Assume the initial displacements and initial velocities of the masses as zero.

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Part VII. Demonstrations

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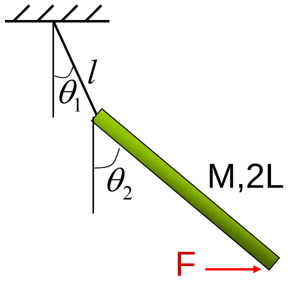
Lagrangian simulation

Advisor : K-S Chen
Speaker : C-X Dai

Model



- Lagrange's Equation

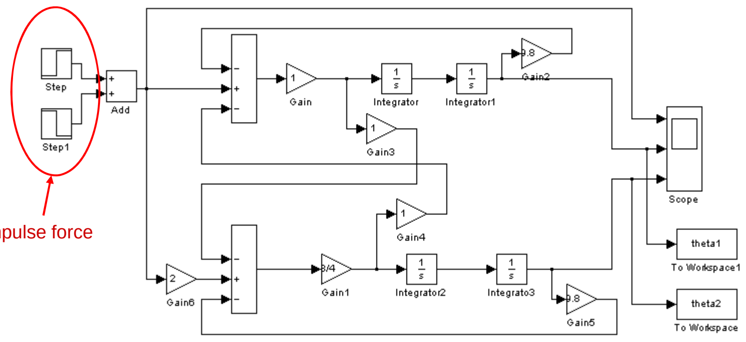
$$\begin{cases} Ml\ddot{\theta}_1 + ML\ddot{\theta}_2\cos(\theta_1 - \theta_2) + ML\dot{\theta}_2^2\sin(\theta_1 - \theta_2) + Mg\sin\theta_1 - F\cos\theta_1 = 0 \\ \frac{4}{3}ML\ddot{\theta}_2 + Ml\ddot{\theta}_1\cos(\theta_1 - \theta_2) - Ml\dot{\theta}_1^2\sin(\theta_1 - \theta_2) + Mg\sin\theta_2 - 2F\cos\theta_2 = 0 \end{cases}$$

- Linearization

$$\begin{cases} Ml\ddot{\theta}_1 + ML\ddot{\theta}_2 + Mg\theta_1 - F = 0 \\ \frac{4}{3}ML\ddot{\theta}_2 + Ml\ddot{\theta}_1 + Mg\theta_2 - 2F = 0 \end{cases}$$

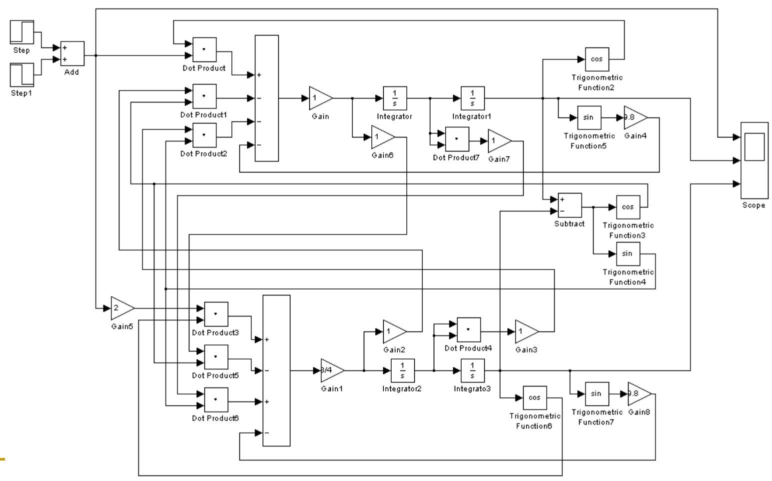
Linear

$$\begin{cases} \ddot{\theta}_1 = \dfrac{1}{Ml}(F - ML\ddot{\theta}_2 - Mg\theta_1) \\ \ddot{\theta}_2 = \dfrac{3}{4ML}(2F - Ml\ddot{\theta}_1 - Mg\theta_2) \end{cases}$$



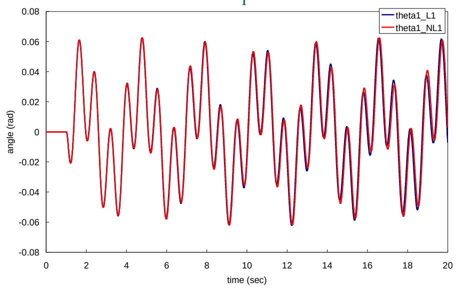
Nonlinear

$$\begin{cases} \ddot{\theta}_1 = \dfrac{1}{Ml}(F\cos\theta_1 - ML\ddot{\theta}_2\cos(\theta_1-\theta_2) - ML\dot{\theta}_2^{2}\sin(\theta_1-\theta_2) - Mg\sin\theta_1) \\ \ddot{\theta}_2 = \dfrac{3}{4ML}(2F\cos\theta_2 - Ml\ddot{\theta}_1\cos(\theta_1-\theta_2) + Ml\dot{\theta}_1^{2}\sin(\theta_1-\theta_2) - Mg\sin\theta_2) \end{cases}$$

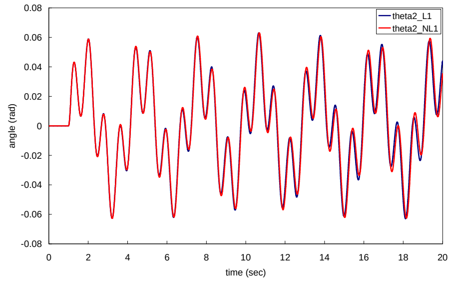


F=1N
Linear and nonlinear comparison

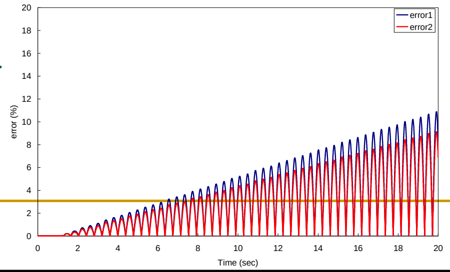
θ_1



θ_2



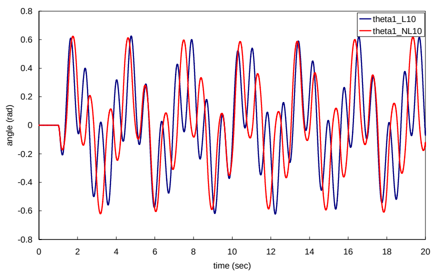
Error



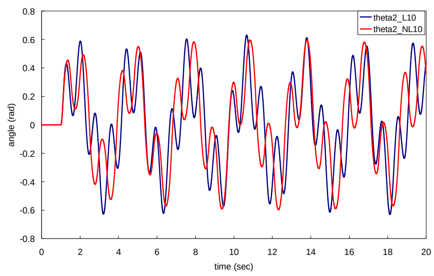
$$\text{Error (\%)} = \frac{\left|\theta_{\text{linear}} - \theta_{\text{nonlinear}}\right|}{\theta_{\text{peak-peak}}} \times 100$$

F=10N

θ_1



θ_2



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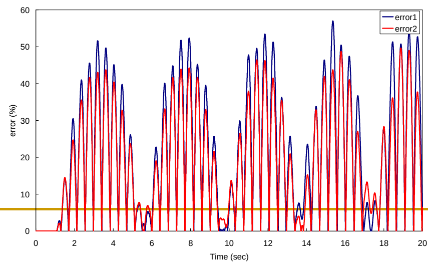
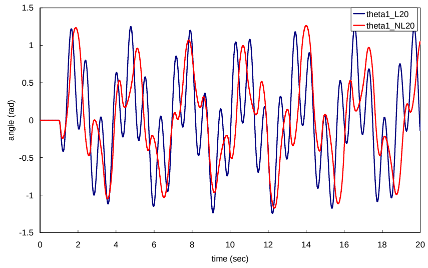
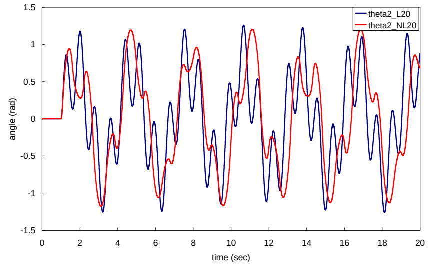


F=20N

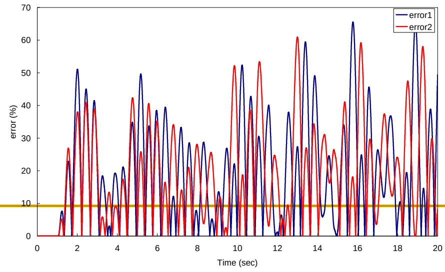
θ_1



θ_2



Error



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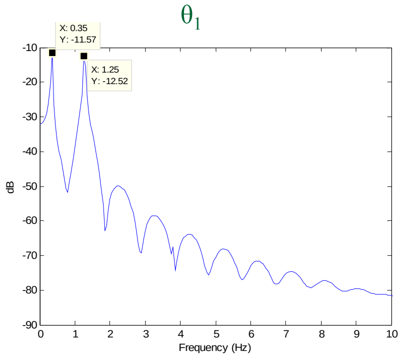
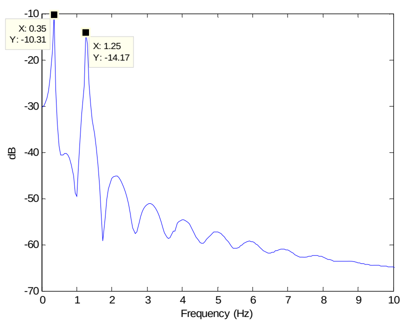
F=1N

Spectrum Analysis (nonlinear)

θ_1



θ_2



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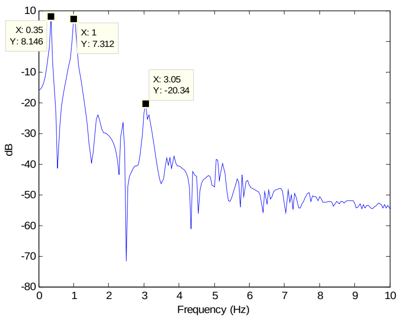
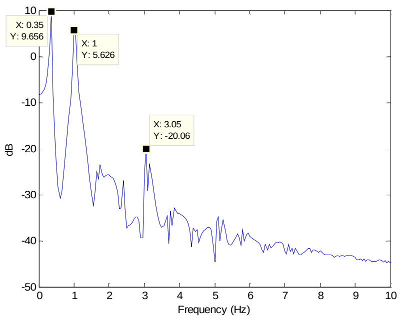
F=10N

θ_1



θ_2



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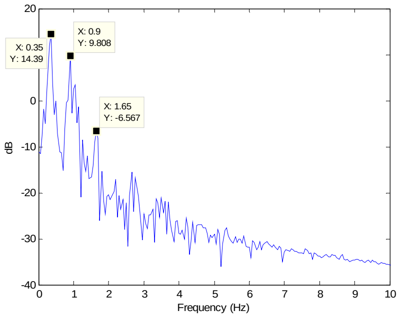
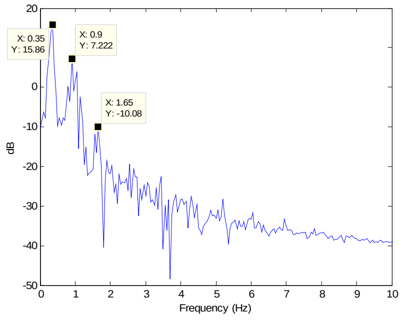
F=20N

θ_1



θ_2



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Conclusion

- When the input force is small, the result from linear model and nonlinear model are very similar.
- When the input force strengthen, the result will be different.

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